

5-4 Multiplying Binomials Mentally

Objective: To find the product of two binomials mentally.

Vocabulary

Quadratic Term A term of degree two. For example, $2x^2$.

Linear Term A term of degree one. For example, $5x$.

Quadratic Polynomial A polynomial whose term of greatest degree is quadratic.
For example, $2x^2 - 5x + 7$.

Example 1 Write $(3x + 1)(4x - 5)$ as a trinomial.

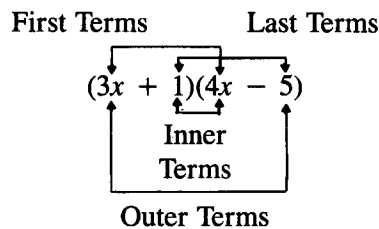
Solution 1 You can work horizontally as shown at the left or vertically as shown at the right.

$$\begin{aligned}(3x + 1)(4x - 5) &= 3x(4x - 5) + 1(4x - 5) \\ &= 12x^2 - 15x + 4x - 5 \\ &= 12x^2 - 11x - 5\end{aligned}$$

$$\begin{array}{r}4x - 5 \\ 3x + 1 \\ \hline 12x^2 - 15x \\ + 4x - 5 \\ \hline 12x^2 - 11x - 5\end{array}$$

Solution 2 Use the FOIL method to multiply in your head.

Think of the products of these terms:



Then write the products:

$$\begin{array}{ccccccc} 12x^2 & - & 15x & + & 4x & - & 5 & = & 12x^2 & - & 11x & - & 5 \\ \text{First} & & \text{Outer} & & \text{Inner} & & \text{Last} & & & & & & \\ \text{terms} & & \text{terms} & & \text{terms} & & \text{terms} & & & & & & \end{array}$$

Write each product as a trinomial.

- | | |
|-----------------------|-----------------------|
| 1. $(x + 6)(x + 1)$ | 2. $(y + 3)(y + 4)$ |
| 3. $(a - 4)(a - 2)$ | 4. $(x - 5)(x - 6)$ |
| 5. $(c + 2)(c + 6)$ | 6. $(k - 3)(k - 6)$ |
| 7. $(a - 3)(a - 7)$ | 8. $(2 + x)(3 + x)$ |
| 9. $(k - 4)(k - 7)$ | 10. $(b - 2)(b + 7)$ |
| 11. $(c - 6)(c + 7)$ | 12. $(a - 4)(a - 6)$ |
| 13. $(2a + 3)(a + 4)$ | 14. $(3x + 2)(x + 4)$ |

5-4 Multiplying Binomials Mentally (continued)

Write each product as a trinomial.

15. $(2a + 7)(a - 2)$

16. $(4a - 1)(3a - 1)$

17. $(3a - 5)(2a - 1)$

18. $(3 - 2a)(2 - 3a)$

19. $(2k + 1)(3k + 4)$

20. $(3x - 2)(x + 5)$

21. $(4x + 3)(2x - 1)$

22. $(7m - 3)(6m + 2)$

Example 2 Write $(3x - 4y)(5x + y)$ as a trinomial.**Solution**

$$\begin{array}{r}
 \begin{array}{c} \text{F} \quad \text{O} \quad \text{I} \quad \text{L} \\
 (3x - 4y)(5x + y) = 15x^2 + (3xy - 20xy) - 4y^2 \\
 = 15x^2 - 17xy - 4y^2 \end{array}
 \end{array}$$

Write each product as a trinomial.

23. $(a - 2b)(a + b)$

24. $(x + 3y)(x + 2y)$

25. $(2x + y)(3x - 2y)$

26. $(3x + y)(x + 2y)$

27. $(4x + y)(2x - 3y)$

28. $(6a - b)(5a - 2b)$

Example 3 Write $(m^2 - 3m)(2m^2 + 5m)$ as a trinomial.**Solution**

$$\begin{array}{r}
 \begin{array}{c} \text{F} \quad \text{O} \quad \text{I} \quad \text{L} \\
 (m^2 - 3m)(2m^2 + 5m) = m^2(2m^2) + m^2(5m) + (-3m)(2m^2) + (-3m)(5m) \\
 = 2m^4 + (5m^3 - 6m^3) - 15m^2 \\
 = 2m^4 - m^3 - 15m^2 \end{array}
 \end{array}$$

Write each product as a trinomial.

29. $(x^2 - 2x)(2x^2 + 3x)$

30. $(a^2 - 3b)(2a^2 + b)$

31. $(u^2 + v^2)(u^2 - 4v^2)$

32. $(a^3 - 3b^3)(a^3 + 4b^3)$

33. $(x^3 - 2x)(x^3 + 1)$

34. $(x^3 - y^2)(3x^3 + y^2)$

Mixed Review Exercises

Simplify.

1. $(3x^2y)(-5x^2y^3)$

2. $(6x^2y^4)^3$

3. $(2n + 3)(3n^2 + n - 2)$

4. $\frac{15r^2 + 20r - 25}{5}$

5. $\frac{(4y)^3}{4y}$

6. $\frac{12 - 6x - 2x^2}{2}$

Solve.

7. $n = 32 - 3n$

8. $3x - (2x + 7) = 7$

9. $4(n + 1) = 3(4 + n)$

10. $5y + 3 = 53$

11. $2(x - 1) - 5 = 9$

12. $5(y - 2) + 4 = 14$